



Modelling the Deflection of Reinforced Concrete Plates Using the Symplectic Method

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Abstract This work aims to predict the flexural behavior of reinforced concrete slabs by combining a smeared finite element model derived from fracture mechanics for the simulation of the decline in the flexural rigidity of the concrete, and the symplectic theory of elasticity, which governs the bending of the structural element in question. Appropriate modifications have been carried out on the smeared finite element model to better reflect the compressive behavior of concrete. The loading is incremental such that in each load step, concrete behavior is treated as an elastic problem, and the response of the concrete slabs determined using the symplectic method, which provides exact bending solutions to thin rectangular plates. For the validation of the model thus proposed, congruent cases of concrete slabs in literature are studied. Evaluation of the results achieved from the proposed model demonstrated relatively good predictability of reinforced concrete slabs in flexure.

Keywords: reinforced concrete slabs, symplectic method, plate deformation, finite element analysis

1. Introduction

The construction industry has experienced rapid growth over the course of the last decade, with an increase in the demand for buildings that are fast to construct, and have large uninterrupted floor areas. The trend towards longer spans and use of lightweight floor systems have the effect of reducing the flexural rigidity of the structural systems [1], bringing to focus the performance of floors subjected to loading. This is especially crucial in reinforced concrete, which undergoes cracking after the yield surface is surpassed, compromising the serviceability and ultimate limit capacities of the structure.

The primary variable under consideration in the response of floors to loading is transverse deformation. There are several analytical solutions developed to determine this deflection for isotropic materials, such as, structural steel. Some of these methods include Navier's Solution and Levy's solution [2]. However, these methods cannot be accurately used to predict the behavior of non-isotropic materials, such as, plain and reinforced concrete. There is need to come up with a creative representation of the material behavior of concrete plates under loading. Such a representation can be developed with the use of numerical methods.



Some of the numerical methods applied in the analysis of plate deformation include finite difference method, finite element method, and finite strip method developed in the late 20th century. More modern methods for the analysis include the method of differential quadrature [3], method of discrete singular convolution [4], differential quadrature element method, and spline element method. Nonetheless, these methods are semi-inverse approaches for Kirchhoff plate bending, and require the pre-selection of a trial function to accurately analyze the plate deformation.

A new symplectic method was developed, which surpasses the limitations of these classical semi-inverse approaches, and extends the scope of analytical solutions [5]. This system implements a systematic and derivational procedure to determine the analytical solutions governed by the various loading and boundary conditions, resulting in a more rational model of the structure compared to the aforementioned numerical methods. This new method is applied in the numerical analysis of the deformation of the concrete plates.

The behavior of reinforced concrete slabs can be assumed to be described by a plane stress field. Kwak and Filippou [6] defined separate models for the concrete and reinforcing steel which were then combined with a model for the bond-slip interaction between the reinforcing steel and the surrounding adjacent concrete, to govern the behavior of the composite element. The failure of the structural element is defined by the rotating smeared crack model, where the crack direction is perpendicular to the direction of the principal tensile strain, and changes with respect to the loading history, thus no shear strain develops in the crack plane, thus negating the need for a cracked shear modulus [7].

In the failure envelope developed by [6], several assumptions were made in the representation of the concrete stress-strain relationship to simplify the model, namely (a) linear elastic up to a stress of $0.6\sigma_{ip}$, (b) linear plastic to $0.6\sigma_{ip}$, (c) linear strain softening, with concrete crushing at $0.85\sigma_{ip}$ where σ_{ip} is the ultimate stress. However, the concrete stress-strain curve determined from experimental studies is not linear, but a curve with the peak at the ultimate stress of the specific design mix. In this study, instead of the above assumptions, a more realistic representation of the concrete stress strain curve is adopted as proposed by Yi et al. [8].

Other nonlinear numerical models have been applied to determine the flexural behavior of non-isotropic structural elements, such as mass-spring models, which was used in

the analysis of plate elements [9] [10], and the stress function method, which was successfully extended to analyze functionally graded anisotropic beams with arbitrary material inhomogeneity along the beam depth [11], [12]. However, these models have several limitations in their application, with the former not scalable to define real life structures, the latter requiring extensive and skillful experience in structural dynamics, and both being limited to specific boundary and support conditions.

The main objective of this work is to verify the application of the symplectic elasticity approach, which has been shown to provide accurate and exact results to bending problems with various loading and boundary conditions [13], to the nonlinear analysis of reinforced concrete slabs by combining it with a modified rotating smeared crack model. The model developed is applied to moderately thin plates, with a length to thickness ratio ranging from 20 to 35. The developed model was validated by two reinforced concrete slabs evaluated experimentally by Taylor et al. [14], and Ghoneim & McGregor [15].

2. Analytical Formulation

2.1 Concrete material matrix

The model developed by Kwak and Filippou [6] is applied with several modifications. The model is chosen due to the following advantages, (a) it possesses increased computational efficiency, (b) it defines the response of concrete structures as dictated by tensile stresses, rather than compressive strength, (c) the flexural behavior is dominated by crack formation and propagation and yielding of reinforcement steel, and (e) the model can be scaled up to real-life structures as it retains objectivity of the results achieved for large finite elements.

The concrete material matrix developed in the model assumes that the material is elastic and homogenous in the macroscopic sense, with the generally non-linear response of concrete divided into 3 stages, resulting in the stress-strain relationship shown in Fig. 1.

Nonetheless, further study on the behavior of concrete under loading has shown that the stress-strain relationship is more accurately represented by a curve, indicating a nonlinear relationship. Yi et al. [8] developed an equation that results in a relatively accurate concrete stress-strain curve, given by:-



$$\frac{f_c}{f_c'} = \frac{\beta_m \left(\frac{\epsilon_c}{\epsilon_c'}\right)}{\beta_m - 1 + \left(\frac{\epsilon_c}{\epsilon_c'}\right)^{\beta_m}}$$

$$\beta_m = \beta_{m,a}(\text{fitted}) = \left[1.02 - 1.17 \left(\frac{E_0}{E_c}\right)\right]^{-0.74} \quad (\epsilon_c \leq \epsilon_c')$$

$$\beta_m = \beta_{m,d}(\text{fitted}) = \beta_{m,a}(\text{fitted}) + (a + bt) \quad (\epsilon_c \geq \epsilon_c')$$

$$a = (12.4 - 1.66 \times 10^{-2} f_{28})^{-0.46}$$

$$b = 0.83 \exp(-911/f_{28}) \quad (1)$$

Where $\beta = \frac{1}{1 - \frac{f_c'}{\epsilon_c' E_{ci}}}$

- f_c concrete stress
- f_c' maximum concrete stress, as f_{28} [16]
- f_{28} concrete strength at 28 days
- ϵ_c concrete strain
- ϵ_c' strain corresponding to max stress
- E_0 secant modulus of elasticity at apex
- E_{ci} initial tangent modulus of elasticity

The initial value of ϵ_c' is assumed to be 0.002 to determine the value of E_0 , and appropriate iterations done to obtain the curve that best represents the problem in question. The limiting concrete strain ϵ_{cf} is taken as 0.0035 [17]. The resulting stress-strain curve for C25 ($f_{28} = 25N/mm^2$) concrete, is shown in Fig. 2.

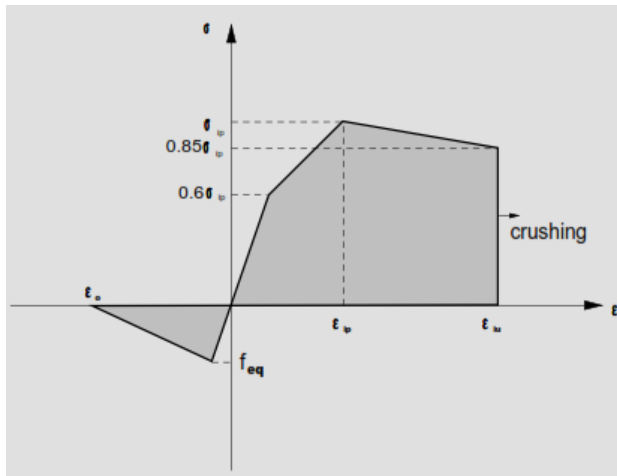


Fig. 1: Piecewise linear stress-strain curve [6]

At the beginning of the loading, the concrete is assumed to be a homogenous, linear isotropic material, defined by the matrix relation:-

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E_{ci}}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2)$$

- Where
- E_{ci} initial tangent modulus
 - ν Poisson's ratio
 - ϵ_x, ϵ_y direct strain
 - γ_{xy} shear strain
 - σ_x, σ_y normal stresses
 - τ_{xy} shear stress

Once the biaxial stress combination exceeds the initial yield surface, the material is assumed to be orthotropic. This assumption holds for all stress values outside the ultimate loading surface. The incremental constitutive relation defining the material at this stage is given by:-

$$\begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\tau_{12} \end{bmatrix} = \frac{1}{1 - \nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} & 0 \\ \nu\sqrt{E_1 E_2} & E_1 & 0 \\ 0 & 0 & (1 - \nu^2)G \end{bmatrix} \begin{bmatrix} d\epsilon_{11} \\ d\epsilon_{22} \\ d\gamma_{12} \end{bmatrix} \quad (3)$$

Where $(1 - \nu^2)G = 0.25(E_1 + E_2 - 2\nu\sqrt{E_1 E_2})$

E_1, E_2 secant moduli of elasticity oriented perpendicular and parallel to crack direction

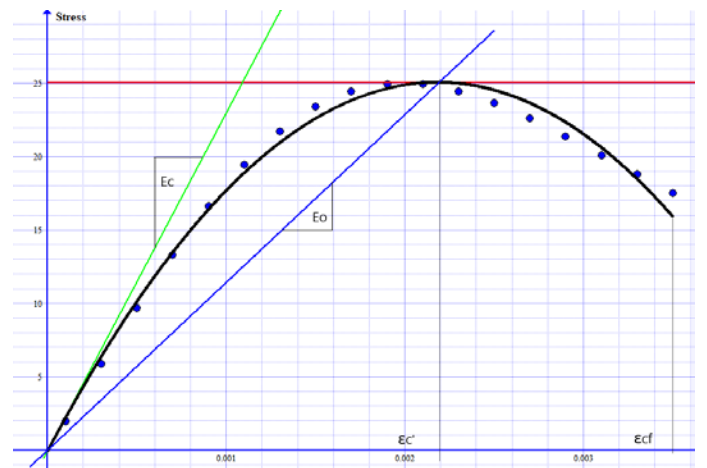


Fig. 2: Nonlinear concrete stress-strain curve

However, it has been shown that the stress-strain relationship is not very sensitive to the compression ratio, thus it can be assumed that $E_1 = E_2$, and as such the model is applicable to plane stress problems [18]. The



concrete Poisson's ratio is also varied with increasing stress following the relation [19]:-

$$v = 0.00189f_t + 0.12 \tag{4}$$

Where f_t tensile stress of concrete
 f_c compressive stress of concrete

2.2 Reinforcement steel material matrix

The reinforcement bars are modelled using the layer model, where the rebars are assumed to be distributed over the concrete element at a certain orientation angle. The rebars embedded in the concrete plate are replaced by an equivalent steel element with distributed uniaxial material properties in each reinforcing direction [20].

The reinforcing steel is assumed to be elastic-perfect plastic in tension and compression, with the axial stiffness considered only in the bar direction. The dimension of the equivalent steel element match those of the concrete element, with the thickness given by Kwak and Filippou [6]

$$t_s = \frac{A_s}{b} = \rho_s d_c \tag{5}$$

where t_s thickness of the steel element
 b spacing of rebars
 ρ_s reinforcing ratio
 d_c effective depth
 A_s cross-sectional area of rebar in a particular direction

The constitutive material matrix is defined by:-

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad \begin{matrix} E = E_{s1} \text{ before yielding} \\ E = E_{s2} \text{ after yielding} \end{matrix} \tag{6}$$

3. Symplectic Elasticity for Thin Plates

Symplectic elasticity is a concept that originated within the Hamilton formulation, where the phase space of certain classical systems have a structure similar to symplectic manifolds, which are closed nondegenerate 2-form geometrical shapes [21]. The symplectic method holds several advantages over other classical methods for the analysis of engineering mechanics, including;-(a) The symplectic method has no need for trial functions, thus applicable to numerous boundary conditions of the plate element, (b) The method consolidates the existing solutions for static mechanics by mapping with a series of zero and nonzero eigen values and their associated

eigenvectors, and (c) The method is applicable to specialized static mechanics problems for which solutions were not previously available [22].

The method is applied to the problem of reinforced concrete slabs as concrete has been shown to exhibit some elasticity in the initial loading stages, also possessing some ductility in the cracked region, and is not completely brittle [23].

Introducing plane elasticity problems into the Hamiltonian canonical equations derives a system of symplectic solution applicable to rectangular thin plates. The symplectic solution can also be derived by an analogy between thin plate bending and plane elasticity, and is applicable to both rectangular and sectorial domains. The governing equation for a thin plate with a uniformly distributed load q is given by

$$\nabla^2_x \nabla^2_y w = \frac{q}{D} \tag{7}$$

where ∇^2 is the Laplace operator, D is the flexural rigidity of the plate element, and w is the transverse deflection of the mid-plane of the plate. The strain energy density in terms of curvature:-

$$\begin{aligned} v_\varepsilon(\kappa) &= \frac{1}{2} \kappa^T C \kappa \\ &= \frac{1}{2} D [\kappa_x^2 + \kappa_y^2 + 2v\kappa_x\kappa_y + 2(1-v)\kappa_{xy}^2] \end{aligned} \tag{8}$$

Where; - v is the Poisson's ratio
 The elasticity coefficient matrix of material:-

$$C = D \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 2(1-v) \end{bmatrix} \tag{9}$$

The curvature vector is given by:-

$$\kappa = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ -\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \tag{10}$$

The Pro-Hellinger-Reissner variation principle for the deformation of thin plates is given by [24]:-



$$\delta \Pi_2 = \delta \left\{ \iint_V [\kappa^T \hat{E}(\nabla) \phi - v_\epsilon(\kappa)] dx dy - \int_{\Gamma_u} (\phi_s \bar{\kappa}_{ns} + \phi_n \bar{\kappa}_s) ds - \int_{\Gamma_\sigma} [\kappa_{ns}(\phi_s - \bar{\phi}_s) + \kappa_s(\phi_n - \bar{\phi}_n)] ds \right\} = 0 \quad (11)$$

Where:-

$$\hat{E}(\nabla) = \begin{bmatrix} \frac{\partial w}{\partial x} & 0 \\ 0 & \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial y} & \frac{\partial w}{\partial x} \end{bmatrix} : \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{1}{2} \left[\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right] \end{Bmatrix} : \phi = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} \quad (12)$$

The subscripts n, s represent the directions normal and tangential to the boundary respectively, while Γ_u and Γ_σ are the corresponding boundaries with specified natural conditions, such as forces and moments, and specified geometric conditions, such as displacements and gradients [24].

Substituting the strain energy density and the curvature vector into the Pro-H-R variation principle equation yields an equation with the state variables ϕ_x, ϕ_y, κ_y and κ_{xy} , whose variation results in the Hamiltonian dual equation:-

$$\dot{v} = H v \quad : \quad v = \{\phi_x, \phi_y, \kappa_y, \kappa_{xy}\}^T \quad (13)$$

The Hamiltonian operator matrix is given by:-

$$H = \begin{bmatrix} \nu & \nu \frac{\partial}{\partial \varphi} & D(1-\nu^2) & 0 \\ -\frac{\partial}{\partial \varphi} & 1 & 0 & 2D(1-\nu) \\ \frac{1}{D} & \frac{1}{D} \frac{\partial}{\partial \varphi} & -\nu & -\frac{\partial}{\partial \varphi} \\ -\frac{1}{D} \frac{\partial}{\partial \varphi} & -\frac{1}{D} \frac{\partial^2}{\partial \varphi^2} & \nu \frac{\partial}{\partial \varphi} & -1 \end{bmatrix} \quad (14)$$

Applying the method of separation of variables to v gives the equation:-

$$v(x, y) = \xi(x)\psi(y) \quad (15)$$

Substituting in the bending strain energy equation yields the expression:-

$$\xi(x) = e^{ux} \quad (16)$$

and the eigenvalue equation:-

$$H\psi(y) = u\psi(y) \quad (17)$$

where u is the eigenvalue in the X-direction and $\psi(y)$ the corresponding eigenvector. The eigenvalues λ in the Y-direction are obtained by the following substitution of the state variables in the eigenvalue equation:-

$$\phi_x = e^{\lambda y} \quad : \quad \phi_y = e^{\lambda y} \quad : \quad \kappa_y = e^{\lambda y} \quad : \quad \kappa_{xy} = e^{\lambda y} \quad (18)$$

Expanding the determinant yields the eigenvalue equation:-

$$(\lambda^2 + u^2)^2 = 0 \quad : \quad \lambda = \pm ui \quad (19)$$

The general solution of the non-zero eigenvalues are expressed as [13]: -

$$\begin{aligned} \phi_x &= A_1 \cos(uy) + B_1 \sin(uy) + C_1 y \sin(uy) + D_1 y \cos(uy) \\ \phi_y &= A_2 \sin(uy) + B_2 \cos(uy) + C_2 y \cos(uy) + D_2 y \sin(uy) \\ \kappa_y &= A_3 \cos(uy) + B_3 \sin(uy) + C_3 y \sin(uy) + D_3 y \cos(uy) \\ \kappa_{xy} &= A_4 \sin(uy) + B_4 \cos(uy) + C_4 y \cos(uy) + D_4 y \sin(uy) \end{aligned} \quad (20)$$

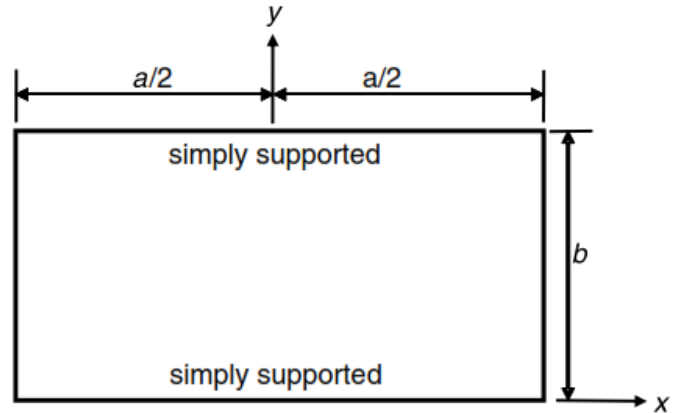


Fig. 3: Coordinate system for plate element

All the constants are not independent and can be determined by choosing one set of constants to be independent, and then substituting the general solution equations into the eigenvalue equation. Furthermore, substituting the general solution into the corresponding boundary conditions on either side of the plate element ($y = b_1$ or b_2), gives the transcendental equation of non-zero eigenvalues and the corresponding eigenvectors. The method of eigenvector expansion is then applied [25].

4. Verification of Results

The validation of any kind of model means to prove the whether the model is true or not, based on a pre-established set of criteria. The validation of the model is an iteration step process composed of conceptual model



validation, computerized model verification, and operational validation [26].

Conceptual model validation determines that the model representation of the problem is ‘reasonable’ for the intended purpose of the model, by checking that the assumptions and theories used to develop the conceptual model are correct. The proposed method applied the compressive stress-strain curve for concrete developed from the equations of Yi et. al. [8] to determine the elastic modulus and Poisson’s ratio of the reinforced concrete plate at different loading intervals. The intervals were chosen such that the curve between each successive point can be assumed to be linear. This then allowed for the application of the symplectic method to get accurate results for the deflection of the reinforced concrete plate.

Computerized model validation, on the other hand, ensures that the implementation of the conceptual model through the computer programming is up to par. This was achieved by ensuring the equations describing the deflection of the plate were accurately programmed into the software. Operational validation is the last step, where the output generated by the simulated model was compared to the problem entity or system, and determine the accuracy of the model to its intended applicability. The operational validity of the proposed model was achieved by comparing the results derived from the proposed model with experimental deflection results conducted on congruent slabs found in literature.

The problem of rectangular reinforced concrete plates simply supported on all four edges is chosen to check the suitability of the proposed model. The plate is bound within the domain defined by $-\frac{a}{2} \leq x \leq \frac{a}{2}$ and $0 \leq y \leq b$ as shown in Fig 3. The boundary conditions of the plate are defined by:-

$$M_y|_{y=0,b} = 0; \quad w|_{y=0,b} = 0; \quad M_x|_{x=\pm\frac{a}{2}} = 0; \\ w|_{x=\pm\frac{a}{2}} = 0; \quad k_y|_{x=\pm\frac{a}{2}} = 0 \quad (21)$$

The bending deflection of the simply supported thin plate under uniformly distributed load using the symplectic method is [13]:-

$$w = \frac{q}{24D} (y^4 - 2by^3 + b^3y) + \\ \frac{2q}{Db} \sum_{n=1}^{\infty} \frac{[\mu_n x \sinh(\mu_n x) - \cosh(\mu_n x) (2 + \alpha_n \tanh \alpha_n)] \sin(\mu_n y)}{\mu_n^5 \cosh \alpha_n} \quad (22)$$

Where $\mu_n = \frac{n\pi}{b}$; $\alpha_n = \frac{\alpha n\pi}{2b}$; $\alpha = \frac{D \times w_{max}}{qb^4}$

The value of α is dependent on the maximum deflection of the plate at a load intensity. For isotropic materials, the value is constant as the flexural behavior is elastic up to

the yield surface. In the case of concrete, however, the value of α varies with increasing load and the cracking propagation. The value adopted in the formula is taken at the beginning of the loading process, where the flexural behavior of concrete is assumed to be linearly elastic.

The Pearson Chi-squared distribution test was applied as a test of the statistical significance of the results from the proposed model. It is the only available statistical test applicable to structural equation modelling to test the goodness of fit of the results derived from the model to those from the experimental tests [27]. The confidence level adopted for this work is 5%.

The chi-square statistic was calculated as [28]:

$$\chi^2 = \sum \frac{(O_o - O_e)^2}{O_e} \quad (23)$$

Where O_o observed outcome
 O_e expected outcome

The chi-squared statistic is then compared to the chi-squared table, and the hypothesis accepted or rejected.

5. Results and Discussions

The reinforced concrete slabs were modelled in finite element software, with pinned linear supports for the edges. The steel reinforcement was modelled as a layer whose thickness and position were determined by the reinforcement placement and quantity provided in the reinforced slab tested in existing literature. The slab element was then divide into smaller finite elements 50mm wide in the X and Y directions as shown in Fig. 4.

The program idealizes the linear pinned support as a series of pinned supports at every node that falls along the edges of the plate. The thickness of the element is taken to be the thickness of the reinforced concrete plate, since no appreciable change in material properties is assumed to occur across the thickness of the plate. The deflection of the plate due to uniformly distributed load is as shown in Fig. 5. The maximum deflection at each load increment was then recorded for future comparison with congruent slabs in literature.

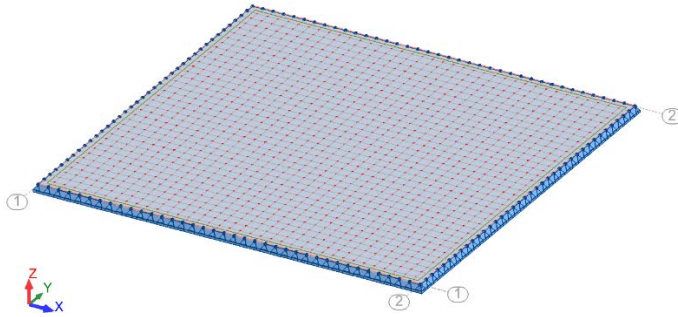


Fig. 4: Finite element division of the RC plate

5.1 Doubly reinforced slab by Ghoneim and MacGregor [15]

An experimental setup on RC slabs subjected to in-plane and lateral loads was carried out by Ghoneim and MacGregor [15], where one of the tested slabs, designated C1, was a square, simply supported along the four edges, and subjected to a uniformly distributed load represented as nine point loads. This slab was analyzed using the proposed model. The slab is a doubly reinforced slab, comprising of two layers of 6.53 mm diameter reinforcement, in both the compressive and tensile zones, to give a reinforcement ratio of 0.38% per layer per direction.

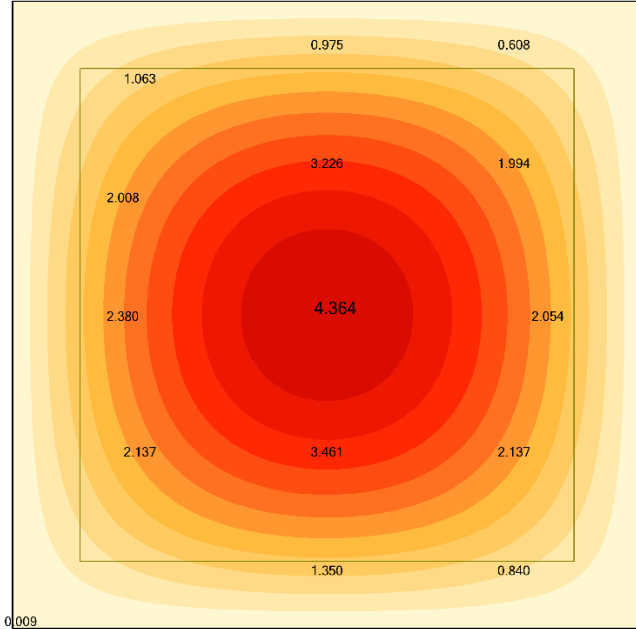


Fig. 5: Deflection distribution on the plate

The geometric properties of the tested slab are given in Table I. The concrete used had a compressive and tensile strength of 25.21 N/mm^2 and 2.31 N/mm^2 respectively. The reinforcement steel had a Young's

modulus of 181.5 kN/mm^2 and an ultimate yield stress of 450 N/mm^2 .

Fig. 6 show the concrete compressive stress-strain curve for the reinforced concrete plate. The load- central deflection curve for the plate developed from the proposed model is plotted in Fig. 7 along with the results from the experimental setup, a layered finite element model proposed by Polak and Vecchio [29], and a layered shear-flexural plate element by Zhang et al. [30].

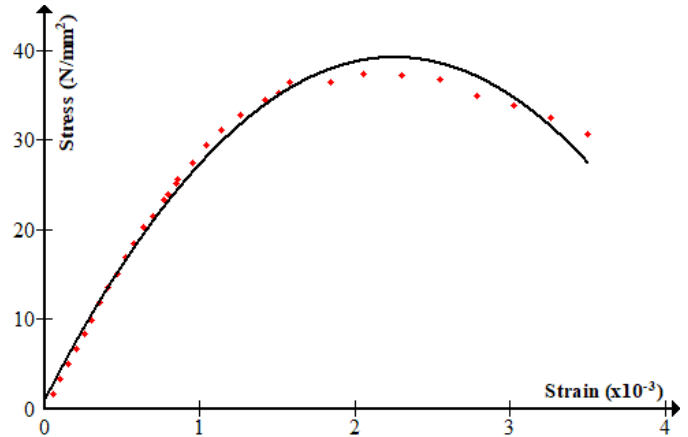


Fig. 6: Stress-Strain curve for Slab C1

Table I: Dimensions of Slab C1 (mm)

Dimensions (mm)	1830 × 1830 × 68
X-Direction, distance of top bar from top surface	21.9
X-Direction, distance of bottom bar from top surface	56.8
Y-Direction, distance of top bar from top surface	15.6
Y-Direction, distance of bottom bar from top surface	50.5

Polak and Vecchio [29] applied the smeared rotating crack approach to account for the reduction in stiffness and strength of concrete due to the presence of cracks in the matrix transverse to the direction of load application. A layered element formulation was then applied to develop a degenerate isoparametric quadrilateral element, with the problems of zero energy and shear locking avoided by selective integration.

Zhang et al [30] proposed a unified displacement-based finite element formulation of a 4-node, 24 DOF rectangular layered plate based on Timoshenko's composite beam functions and the Mindlin-Reissner thick plate theory. The composite beam functions proposed by Timoshenko were extended to the analysis of reinforced concrete plates avoiding the problem of shear locking. The element was formulated by employing a Total Lagrangian approach and incorporated into a nonlinear finite element solution algorithm.

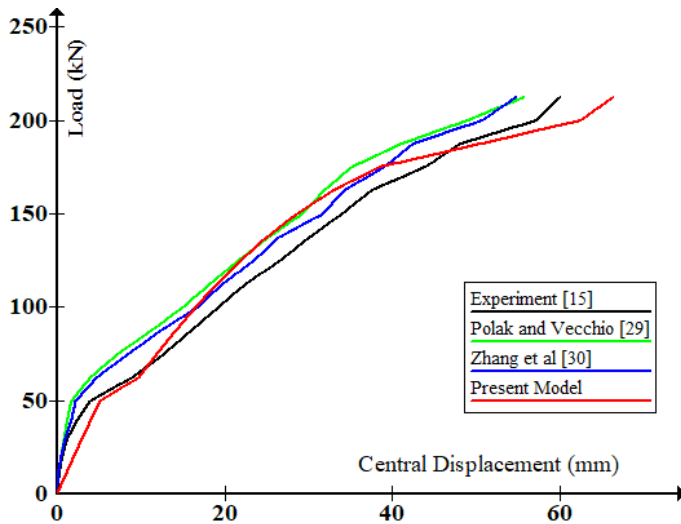


Fig. 7: Load deflection curve for Slab C1

5.2 Singly reinforced slab supported by its four edges

A square, simply supported slab subjected to uniformly distributed loading was tested by Taylor et al. [14]. The dimensions of the slab were $1980 \times 1980 \times 51\text{mm}$, with roller supports placed 75mm from the edges, resulting in an effective span of $1830 \times 1830 \times 51\text{mm}$. The slab had a single layer of 4.76mm diameter reinforcement in the tensile zone, with the reinforcement in the x- direction spaced at 76mm at an effective depth of 43.6mm, and that of the y-direction spaced at 63.5mm at an effective depth of 39mm.

The concrete used had a compressive strength of 35.04 N/mm^2 and a tensile strength of 3.6 N/mm^2 respectively. The reinforcement steel had a Young's modulus of 206.91 kN/mm^2 and an ultimate yield stress of 375.9 N/mm^2 . The decline in the elastic modulus of the plate with increasing strain due to load is shown in Fig. 8. Lima et al [31] combined the Mazars damage model to simulate stiffness reduction in concrete post-cracking with the Classical Theory of Laminates to predict the flexural behavior of reinforced concrete slabs. The load-deflection curve obtained from the proposed model is compared to results obtained from the experimental setup and the simplified isotropic damage model proposed by Lima et al [31] in Fig. 9.

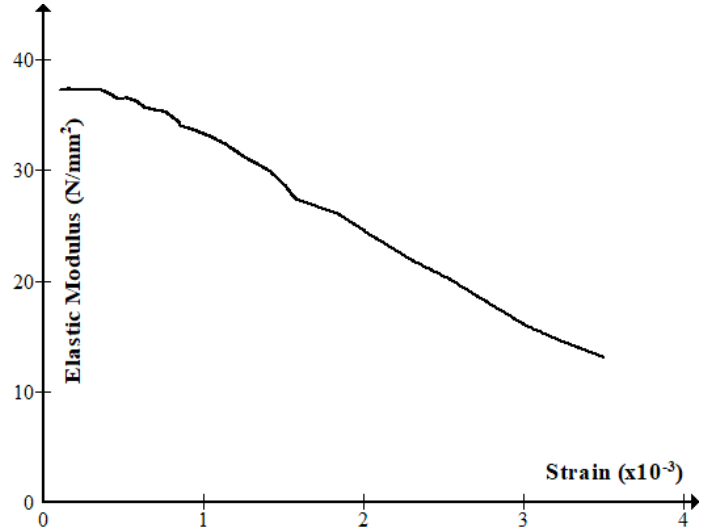


Fig. 8: Plot of elastic modulus against strain for singly reinforced concrete slab

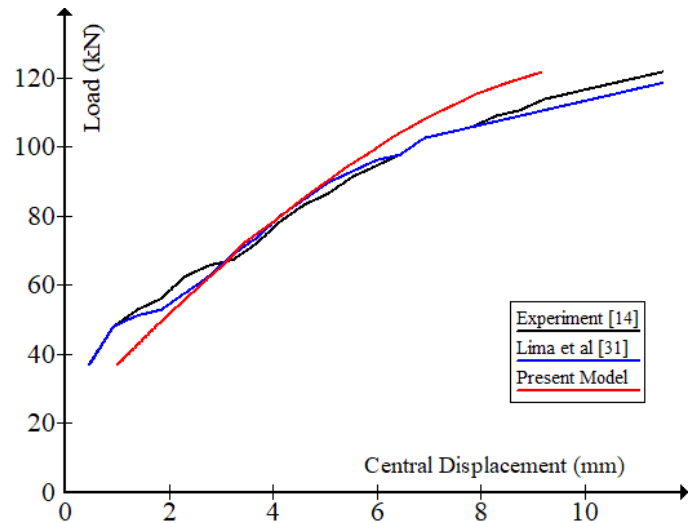


Fig. 9: Load deflection curve of a singly reinforced simply supported plate

6. Conclusion

Based on the modified stiffness approach and layer approach for modelling of reinforced concrete slabs, idealized stress-strain relations are applied to virtual concrete and reinforcement steel layers, for which the symplectic method is applied to obtain accurate deflection values of moderately thin reinforced concrete slabs.

The proposed model is able to overcome the orthotropic, non-homogenous nature of reinforced concrete plates by dividing the problem into smaller intervals, where the flexural behavior can be defined as an elastic problem. The efficiency of the proposed model to predict the load-deflection behavior of reinforced



concrete slabs is exhibited by the comparison with experimental results and previous models, providing relatively acceptable results.

The results of the deflection of the doubly reinforced slab differ from the experimental results with a standard deviation of 3.62 mm, with the largest difference observed being 6.37 mm towards the failure point. The chi-squared analysis of the results yielded a value of 15.82, with a degree of freedom of 18, thus yielding a probability of 0.604.

For the singly reinforced slab, the results from the proposed model differ from the experimental results with a standard deviation of 1.173 mm, with the largest difference measured being 2.32 mm. The chi-squared analysis of the results yielded a value of 5.189, with a degree of freedom of 24. There is therefore, no statistically significant difference between the results from the model and those from the experimental set ups for both doubly and singly reinforced concrete slabs.

It is also observed that the proposed model offers slightly more accurate results than some other numerical solutions for similar static mechanics problems. The proposed method has accuracy similar to the layered shear-flexural plate element presented by Zhang et al [30] for the doubly reinforced slab, which gave a standard deviation of 3.58 mm. It also offers slightly more accurate results than the layered finite element model proposed by Polak and Vecchio [29], whose standard deviation is 4.92mm. The proposed model also offers slightly greater accuracy than the simplified isotropic damage model proposed by Lima et al [31] for the singly reinforced slab, giving a standard deviation of 2.4mm.

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